AN ANALYTICAL EVALUATION OF THE IMPACT OF OPENNESS ON ECONOMIC PERFORMANCE: A THREE-SECTOR GENERAL EQUILIBRIUM OPEN ECONOMY MODEL

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Abstract
The theoretical linkages between openness and long-run economic growth are established with the development of new growth theories. In this literature, it has been shown that openness affects economic growth through a number of channels and the direction of this effect is closely related to the complex interactions exist among sectors of the economy. The theoretical models on the subject are very sophisticated and provide useful insights into the linkages between openness and growth but the complexity of these models makes it difficult to identify the contribution of a particular link to economic performance. To make it easier to understand the interactions among sectors of the economy, and to provide consistency with the conventional sectoral classification employed in economics, in this article, we provide an illustrative three-sector open-economy general-equilibrium model. Using this model, we will show analytically the impact of openness on sectors of the economic and identify the channels and magnitude of this effect. We will also support our analytical findings by providing the results obtained from simple simulation exercises.

Key Words: Openness, Economic Reform, General Equilibrium, Sectoral Interactions.

1. Introduction
The theoretical literature on the relationship between openness and economic performance shows that there are complex interactions among sectors of the economy and these interactions have important implications for the impact of openness on economic performance1. The theoretical models on the subject are very sophisticated and provide useful insights into the linkages between openness and growth but the complexity of these models makes it difficult to identify the contribution of a particular link to economic performance. The characteristics of the sectors used in these models are also different from the conventional classification of sectors in macroeconomics. To make it easier to understand the interactions among sectors of the economy, and to provide consistency with the conventional sectoral classification employed in economics, in this article, we provide an illustrative three-sector open-economy general-equilibrium model.

An examination of the theoretical literature on the subject shows that the investigation of policy reform centred around the possible direct and indirect impact of policy change on output, prices, and international trade. The indirect effects of policy reform involve the impact of policy change on technology and increased capital productivity. Although an empirical analysis is generally carried out at the aggregate level, the theoretical justification of policy reform is based at the sectoral level. In this respect, it seems vitally important to understand the interaction among sectors of the economy to understand the effect of policy change on different aspects of the economy.

In this context, the general equilibrium analysis is very useful because of the analytical richness it provides and because it allows us to discuss important aspects of economic reform programmes within a unifying framework. Furthermore, it facilitates an explicit illustration of complex interactions among sectors of the economy. Considering the fact that the theoretical justification for a reform programme is mainly based on the neo-classical framework, it also seems reasonable to start an analytical evaluation of the economic reform programme by making use of a general equilibrium-modelling framework.

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To make it easier to understand the interactions among sectors of the economy, and to provide some insight in understanding policy changes, an illustrative three-sector open-economy general-equilibrium model is presented in the following section. However, it is worth noting that the main aim of this model is to demonstrate the importance of sectoral interactions in the reform process rather than to provide a comprehensive model of economic reform that can be used in empirical analysis. However, the model can serve as a framework for thought about the reform programme. Furthermore, it motivates the use of sectoral data on output, exports, imports, and prices in empirical analysis of openness-growth relationship.

The effect of policy changes, technological change, and the change in capital stock on output, international trade and prices will be examined within this framework. Section 2 presents the three-sector open-economy model. It is assumed that there are two consumption goods sectors and one capital goods sector. The latter uses imported inputs as an additional factor of production in the production process. The subsequent three sections discuss the effect of economic policy, technology, and changes in capital stock on different variables of the economy. While Section 3 illustrates the effect of policy change on sectoral prices, wages and rental rate, Section 4 shows the impact of economic policy on sectoral output levels, and Section 5 provides discussion of the impact on exports and imports. Section 6 concludes. The detailed derivation of the three-sector open economy model is provided in the Appendix to this chapter.

2. INTERACTIONS AMONG SECTORS OF THE ECONOMY: A THREE-SECTOR OPEN-ECONOMY MODEL

The purpose of this section is to introduce a three-sector open economy general equilibrium model. It is evident that the implications of policy instruments and the results of interactions in the economy are closely related to the characteristics of the model build up. It is equivalently true that an analytical demonstration of sectoral interactions makes it easier to understand the implications of policy changes. In this way, an appropriate model can easily illustrate what happens to relevant variables under policy changes and provide some insights about the effects of interventions on the performance of the economy. Considering the fact that the economy as a whole involves complex interactions of firms and individuals, it is not possible to formulate every action that is going on in the economy in one model. Therefore, this section is restricted to a three-sector model of an open economy. The model can easily be generalised to many sectors, but this does not add more to the understanding of the implications of the model.

The model we present below is an extended version of a baseline general equilibrium model. The structure of standard general equilibrium models are presented by Simpson (1975), and Dinwiddiy and Teal (1988), who provide a good discussion about the structure of general equilibrium models and give some numerical examples. Hazari et.al. (1981) and Hazari (1978) also show the effects of trade distortions in a general equilibrium modelling framework. In this sense, our model follows the standard general equilibrium modelling framework. However, it differs from the models presented in the literature in terms of the assumptions related to the characteristics of sectors, and specific forms that utility and productions functions take. This allowed us to find the equilibrium values of the endogenous variables of the model.

To this end, it is assumed that there are three sectors in the economy: two consumption goods sectors, and one capital goods sector. The types of sectors of the model are selected to represent a sample of the sectors available to an economy. The first two sectors of the economy use labour and capital in production. While the first consumption goods sector exports part of its output, the total output of the second sector is consumed domestically. The third sector produces capital goods for the other sectors as well as for itself using labour, capital and imported inputs.

The main equations of the full general equilibrium open economy model are presented below in Table 2.1. The full derivation of the model is also provided in the Appendix to this chapter. In there, we explicitly show the derivation of the equations (2.1) to (2.24) presented in Table 2.1.

Table 2.1 Three-Sector Open Economy Model

<table>
<thead>
<tr>
<th>Commodity Markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
</tr>
<tr>
<td>$C_i = \theta(1-s)Y / P_i$</td>
</tr>
</tbody>
</table>
\[ C_1 = (1 - \theta)(1 - s)Y / P_1 \]  
\[ I = sY / P_1 \]  
\[ \text{(2.2)} \]
\[ \text{(2.3)} \]

\text{Unit Prices}
\[ P_1 = \frac{w^r \cdot r}{A_1} \left[ \frac{1}{\alpha^{-1}} \right] \]  
\[ P_2 = \frac{w^r \cdot r}{A_2} \left[ \frac{1}{\delta^{-1}} \right] \]  
\[ P_3 = \frac{w^r \cdot r \cdot P_1^{1 - \beta_3 \cdot \beta_1}}{A_3} \left[ \left( \frac{1}{\beta_1} \right)^{\beta_3} \left( \frac{1}{\beta_3} \right)^{\beta_3} \left( \frac{1}{1 - \beta_1 - \beta_3} \right)^{\beta_3 \cdot \beta_1} \right] \]  
\[ \text{(2.4)} \]
\[ \text{(2.5)} \]
\[ \text{(2.6)} \]

\text{Market Clearing}
\[ C_1 = Y_1 - X \]  
\[ C_2 = Y_2 - G \]  
\[ I = Y_3 \]  
\[ \text{(2.7)} \]
\[ \text{(2.8)} \]
\[ \text{(2.9)} \]

\text{Factor Markets}

\text{Demand}
\[ L_1 = Y_1 \left( \frac{w}{1 - \alpha} \right) \left( \frac{r}{\alpha} \right)^\alpha \]  
\[ L_2 = Y_2 \left( \frac{w}{1 - \delta} \right) \left( \frac{r}{\delta} \right)^\delta \]  
\[ L_3 = Y_3 \left( \frac{w}{\beta_1} \right) \left( \frac{r}{\beta_1} \right)^\beta_3 \left( P / (1 - \beta_1 - \beta_3) \right)^{\beta_3} \]  
\[ K_1 = Y_1 \left( \frac{w}{1 - \alpha} \right) \left( \frac{r}{\alpha} \right)^{\alpha-1} \]  
\[ K_2 = Y_2 \left( \frac{w}{1 - \delta} \right) \left( \frac{r}{\delta} \right)^{\delta-1} \]  
\[ K_3 = Y_3 \left( \frac{w}{\beta_1} \right) \left( \frac{r}{\beta_1} \right)^{\beta_3-1} \left( P / (1 - \beta_1 - \beta_3) \right)^{\beta_3} \]  
\[ M = Y_3 \left( \frac{w}{\beta_1} \right) \left( \frac{r}{\beta_1} \right)^{\beta_3} \left( P / (1 - \beta_1 - \beta_3) \right)^{\beta_3} \]  
\[ \text{(2.10)} \]
\[ \text{(2.11)} \]
\[ \text{(2.12)} \]
\[ \text{(2.13)} \]
\[ \text{(2.14)} \]
\[ \text{(2.15)} \]
\[ \text{(2.16)} \]

\text{Market Clearing}
\[ L_1 + L_2 + L_3 = \bar{L} \]  
\[ K_1 + K_2 + K_3 = \bar{K} \]  
\[ \text{(2.17)} \]
\[ \text{(2.18)} \]

\text{Consumer's Income}
\[ Y = w(L_1 + L_2 + L_3) + r(K_1 + K_2 + K_3) \]  
\[ \text{(2.19)} \]

\text{Public Sector}

\text{Tax Revenue}
\[ T = eP_1 \tau M \]  
\[ \text{(2.20)} \]

\text{Budget Constraint}
\[ G = T \]  
\[ \text{(2.21)} \]

\text{Foreign Sector}

\text{Price Equations}
\[ P_i = eP' \]  \hspace{1cm} (2.22)
\[ P = eP' (1 + \tau) \]  \hspace{1cm} (2.23)

Balance of Payments constraint
\[ P'_1 X - P' M = 0 \]  \hspace{1cm} (2.24)

While \( C_i, P_i, Y_i, L_i, \) and \( K_i \) represent the amount of consumption, output, labour, capital, and prices in sector \( i \) respectively, \( P, w, \) and \( \tau \) represent the price of imported input, the wage rate and the rental ratio respectively. \( i = \) sectors 1, 2, and 3. The other variables, \( I, G, M, X, T, e, \) are the amount of investment, government expenditure, the amount of imports, exports, the total tax revenue and exchange rate respectively.

Equations (2.1) – (2.9) represent commodity markets. The consumer spends a fixed amount of his income on consumption goods 1 and 2, and saves the rest. While equations (2.1) and (2.2) give the consumer’s demand for goods 1 and 2 that stem from the consumer’s maximisation problem, equation (2.3) shows the demand for capital goods, which is a function of total savings. Because constant returns are assumed in production, there are no supply functions. Instead, the equilibrium in production is given by unit prices and shown by equations (2.4) to (2.6). Equations (2.7) to (2.9) represent market-clearing conditions in commodity markets. According to these equations, the market clears when domestic consumption plus exports are equal to the supply of sector 1, when government expenditure plus domestic consumer demand are equal to the supply of sector 2 and when the demand for capital goods is equal to its supply.

Equations (2.10) to (2.16) illustrate the conditional demand functions that come from firms’ profit maximisation problems and zero profit conditions. In factor markets, the market clears when the total demand for labour and capital is equal to the stock of capital and labour (2.17 to 2.18). Consumer income is equal to total wages and rental income and presented in equation (2.19). In this model, the government collects tariffs from imports and spends its revenue on the second consumption good. Total tariff revenues and the government’s budget constraint are given in equations (2.20) to (2.21). Under the small country assumption, the prices of exports and imports are taken as given and defined by equations (2.22) to (2.23). The model is closed by the balance of payments equation (2.24).

In summary, there are 24 endogenous variables in the model and these variables are \( C, C_i, I, P_i, P', w, r, Y_i, Y_i, L_i, L_i, K_i, K_i, G, M, X, T, e \). The exogenous variables of the model are \( L, K, P', P', s, \) and \( \tau \). They represent the stock of labour and capital, the foreign price of the imported intermediate goods and export-goods, the saving ratio, and the tariff rate respectively. To solve the model, the exchange rate is normalised at unity, \( e = 1 \).

Completing the general description of the model, the following three sections provide the effect of change in tariff rates, technology and capital stock on prices, output and international trade in turn. The analytical results on these subjects will be supported with simple simulation exercises. It is worth noting that we preferred to study the effects of policy on prices before output in this chapter. This is because providing discussion about prices at first significantly simplifies the discussions about the effects of policy on output and international trade.

3. THE IMPACT OF ECONOMIC POLICY, TECHNOLOGY AND CAPITAL STOCK ON PRICES

In this section, the implications of a percentage change in tariffs, technology, and capital stock are examined for factor and commodity prices. First, the implications of a small increase in tariff rate will be considered for the wage and rental rate and commodity prices \( P_i \) and \( P' \). The examination of the \( P' \) is excluded because it was assumed constant following the small country assumption. Then, the same exercises will be repeated for a small increase in technology and capital stock.
Effect of tariff

The wage response can be obtained by taking the logarithm and then differentiating the equilibrium wage equation (A.52) given in the Appendix to this chapter with respect to tariffs. Hence, by differentiating equation (A.52) and (A.53) with respect to \( \tau \), we obtain:

\[
\frac{1}{w} \frac{\partial w}{\partial \tau} = \frac{\alpha (\alpha - \delta) s (1 - \beta_1 - \beta_2)}{(1 + \tau)^2 \Psi (1 - \Psi)} \tag{3.1}
\]

\[
\frac{1}{r} \frac{\partial r}{\partial \tau} = \frac{(\alpha - 1) (\alpha - \delta) s (1 - \beta_1 - \beta_2)}{(1 + \tau)^2 \Psi (1 - \Psi)} \tag{3.2}
\]

where \( \alpha \) and \( \delta \) are the coefficients on capital in sector 1 and 2’s production functions; hence they represent capital intensities in these sectors respectively. While \( s (1 - \beta_1 - \beta_2) \) shows the share of imported inputs in total production, \( \Psi \) and \( (1 - \Psi) \) represent functions of the parameters of the model.

Equations (3.1) and (3.2) show that a small increase in tariff rates may lead to an increase (or decrease) in wage rate and decrease (or increase) in rental rates depending on the relative capital intensity of sector 1 and 2. If sector 1 is relatively more capital intensive than sector 2 \( (\alpha > \delta) \), wages will increase and the rental rate will decrease unambiguously in response to a small change in the tariff rate or vice-versa. This is easy to understand once we remember that the output of sector 1 is partly exported and a rise in tariff revenue increases the demand for output in sector 2. If the tariff is increased, and sector 1 is relatively more capital intensive, the balance of payments constraint requires exports to decrease. As a result, the production in sector 1 decreases and capital and labour are released from this sector. However, since the proportion of capital and labour released from sector 1 is different from the capital-labour proportions demanded by sector 2, the price of capital will decrease and the price of labour will increase. The magnitude of proportional changes that occur in wage and rental rates depends on the share of imported inputs in total production.

In a similar way, the proportional change in commodity prices can be examined for a small change in tariffs. As shown in equations (2.5) and (2.6), commodity prices \( P_1 \) and \( P_2 \) are functions of wage and rental rates. We, therefore, can find the effect of tariffs on commodity prices by taking logarithms and then derivatives of equations (2.5) and (2.2) with respect to tariffs because we already know the effect of tariffs on wages and rental rates from equations (3.1) and (3.2). Therefore, the proportional change in prices in response to a small change in tariffs will be:

\[
\frac{1}{P_2} \frac{\partial P_2}{\partial \tau} = \frac{(\alpha - \delta) s (1 - \beta_1 - \beta_2)}{(1 + \tau)^2 \Psi (1 - \Psi)} > 0 \tag{3.3}
\]

\[
\frac{1}{P_3} \frac{\partial P_3}{\partial \tau} = \frac{(1 - \beta_1 - \beta_2) [s (\alpha - \delta) (\alpha \beta_2 + \alpha \beta_1 - \beta_1) + (1 + \tau) \Psi (1 - \Psi)]}{(1 + \tau)^2 \Psi (1 - \Psi)} > 0 \tag{3.4}
\]

Equations (3.3) and (3.4) indicate that a small increase in tariff rates unambiguously increases commodity prices \( P_2 \) and \( P_3 \). In the first equation, the square of the difference between the capital intensities of sector 1 and 2 enters into the equation and the sign of the function is obviously positive. The sign is also positive in the second equation because the first term in brackets is always a very small number and smaller than the second term. Intuitively, an increase in tariff rates increases government revenue and hence demand for sector 2’s output and the price of this commodity. Since intermediate imported inputs enter into the production function of sector 3 as an additional factor of production, the cost of production in this sector unambiguously rises. Although it is difficult to forecast the exact magnitude of the change in \( P_3 \), it is closely related to the shares of labour, capital, and especially imported inputs, used in production. These shares are represented by \( \beta_2 \), \( \beta_1 \) and \( (1 - \beta_1 - \beta_2) \).

Technological change, factor rewards, and commodity prices
As mentioned in the literature review section, one of the channels through which openness affects economic performance is the positive effect of openness on technological change. Openness may induce technological advances through learning from foreign trade partners, increased competition, and technology transfer. In this sense, it is important to understand the implications of technological change in one sector for other sectors of the economy. The purpose of this subsection is to examine the implications of a small change in technology for factor prices and commodity prices. The following sections will consider the effect of technology on output levels and international trade.

The level of wages that satisfy general equilibrium in the economy is given in equation (A.5.2) in the Appendix to this chapter. As shown in this equation, wages are a function of the capital-labour ratio, parameters of the model and the level of technology only in sector 1. This may seem surprising at first but it is the direct result of the exogenous nature of the technology and general equilibrium modelling. This point will be clearer once we consider the effect of technological change on commodity prices.

Taking the derivative of wage and rental rate equation with respect to technology, we obtain:

\[
\frac{\partial w}{\partial A_1} = P_1 \alpha^a (1 - \alpha)^{1-a} \left( \frac{K\Psi}{L(1 - \Psi)} \right) > 0 \tag{3.5}
\]

\[
\frac{\partial r}{\partial A_1} = P_1 \alpha^a (1 - \alpha)^{1-a} \left( \frac{K\Psi}{L(1 - \Psi)} \right) > 0 \tag{3.6}
\]

As the equations (3.5) and (3.6) indicate, a small change in technology in the export sector increases wage and rental rates in the economy. The observed symmetry among these equations implies that technological change has no effect on the wage rental ratio. The reason for this seeming contradiction will be explained after considering the effect of technological change on commodity prices. For this purpose, the equilibrium commodity prices given in equations (A.63) and (A.64) in the Appendix to this chapter are reproduced as,

\[
P_2 = \frac{\alpha^a (1 - \alpha)^{1-a} \left( \frac{K\Psi}{L(1 - \Psi)} \right)^{a-d}}{\delta^d (1 - \delta)^{-d}} \tag{3.7}
\]

\[
P_3 = \frac{\alpha^a (1 - \alpha)^{1-a} \left( \frac{K\Psi}{L(1 - \Psi)} \right)^{a-b}}{\beta^b \beta^c} \cdot (1 - \beta_1 - \beta_2)^{\gamma - \beta} \tag{3.8}
\]

As equations (3.7) and (3.8) imply, while a small change in own technology decreases commodity prices, changes in technology in the export sector increases these prices. Combining these results with wage and rental rate responses to technological change, we can explain why technological change, other than that in the export sector, has no effect on wages or the rental ratio. As mentioned before, the small economy assumption requires constant export and import prices. When sector 1’s technology changes, output in this sector increases and, hence, wage and rental rates as well because the commodity price \(P_1\) is constant. In equilibrium, this leads to an overall increase in wages and rental rates in the economy as shown in equations (3.5) and (3.6).

However, this is not true for technological change in other sectors because commodity prices in those sectors decrease in response to own technology. To be more precise, let us assume a small change has occurred in the level of technology in sector 2. On the one hand, this unambiguously increases production in this sector; on the other hand, the commodity price in this sector decreases leaving the nominal value of output constant. Since a zero profit assumption requires a nominal value of output equal to payments to factors of production, there is no reason why wage and rental rate should rise. In other words, an increase in output cannot be transferred to the factors of production because it is compensated by an equivalent decrease in prices. Hence, a rise in output is transferred to, and consumed by, consumers through lower prices. In summary, technological change has no effect on factor prices when commodity prices are flexible in the general equilibrium model.

The Impact of capital accumulation on factor rewards, and commodity prices
Although there is no established direct link between openness and capital accumulation, it is argued that openness increases capacity utilisation and encourages the efficient use of capital. Thus, in relation to our model, the fact that openness removes excess capacity may be interpreted as an increase in capital stock.

In the rest of this subsection, the implications of a small change in capital stock for wage and rental rates and commodity prices will be considered.

To obtain the response of wage and rental rates to a small change in capital stock, we can take the derivative of wage and rental rate equations (A.52) and (A.53) with respect to capital stock. This provides the following result,

\[
\frac{\partial w}{\partial K} = \alpha P \alpha^\prime(1-\alpha)^{-\alpha} K^{-\alpha+1} \left( \frac{\Psi}{L(1-\Psi)} \right)^{\alpha} > 0 
\]

(3.9)

\[
\frac{\partial r}{\partial K} = (\alpha -1) P \alpha^\prime(1-\alpha)^{-\alpha} K^{-\alpha+1} \left( \frac{\Psi}{L(1-\Psi)} \right)^{\alpha-1} < 0 
\]

(3.10)

As given in equations (3.9) and (3.10), while a small change in capital stock reduces the price of capital because of an increase in its supply, it increases wage rates creating excess demand for labour. Thereby, the wage rental-ratio is always positive. The magnitude of the change is closely related to the capital intensity of capital in sector 1 and is a direct result of a small country assumption where commodity price \( P_1 \) is constant. To clarify this point, we take the log-derivative of equation (2.4) with respect to capital stock. This provides, after some manipulation, \( \delta \ln w = \frac{-\alpha}{(1-\alpha)} \delta \ln r \). This equation says that a one percent increase in the rental rate results in a more than one percent decrease in wages if the share of capital (\( \alpha \)) is higher than labour (\( 1-\alpha \)) or vis-à-vis.

However, the implications of a small change in capital stock for commodity prices are not so clear. Taking the logarithm and then the derivatives of price equations (3.7) and (3.8) with respect to capital stock, we obtain:

\[
\frac{1}{P_0} \frac{\partial P_0}{\partial K} = \alpha - \beta 
\]

(3.11)

\[
\frac{1}{P_i} \frac{\partial P_i}{\partial K} = \alpha(\beta_2 + \beta_1) - \beta_i 
\]

(3.12)

Equations (3.11) and (3.12) show that a small change in capital stock leads either to a negative or positive percentage change in commodity prices depending on whether the export sector is relatively more capital intensive than other sectors or not. The multiplication of \( \alpha \) by \( \beta_i + \beta_2 \) makes it possible to compare capital intensities between the export sector and sector 3 because of the existence of three factors of production in the latter sector. Intuitively, a proportional increase in wages needs to be higher than a proportional decrease in the rental rate to keep the price of export sector constant when the share of capital is higher in production as shown above. Furthermore, if the export sector is relatively more capital intensive than sector 2, then the price of commodity 2 must increase to comply with the given relationship between wage and rental rates above. The same argument holds for the capital goods sector as well. Simply, commodity prices of sector 1 and 2 increase (decrease) if the export sector is relatively more (less) capital intensive than sector 1 and 2.

**Simulation Results**

Until now, we have examined analytically the implications of change in tariff, technology, and capital stock for factor and commodity prices. This section will provide numerical examples of these implications. For given values of parameters of the model and exogenous variables, the effects of a change in the tariff rate, capital stock and technology have been considered and provided in Table 3.1. The implications of these changes have been examined under two different scenarios. In the first, we
assumed that the export sector is relatively more capital intensive than the second sector. In the second scenario, the reverse of the first is examined. The result of both scenarios has been provided in the first and the second part of the table respectively. The first row of both parts of the table provides these values for each variable. The values of parameters and exogenous variables are given in the note to the table.

As presented in the table, while the wage rate increases following an increase in the rate of tariff, the level of capital stock and the level of technology in sector 1, and the technological advance in sectors 2 and 3 has no effect on wages. However, when the capital intensities are changed, as shown in the second part of the table, an increase in the tariff rate reduces wages. We are not going to provide all the results presented in the table but the inspection of the rest of the table will confirm the results given in the analytical sections.

Table 3.1. The effect of a change in tariff rate, technology and capital stock on commodity prices, wages, rental rates and wage rental ratio

<table>
<thead>
<tr>
<th></th>
<th>wage rate</th>
<th>rental rate</th>
<th>w / r ratio</th>
<th>P_2</th>
<th>P_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Model</td>
<td>0.798</td>
<td>1.329</td>
<td>0.600</td>
<td>1.427</td>
<td>2.471</td>
</tr>
<tr>
<td>Tariff=0.2</td>
<td>0.802</td>
<td>1.326</td>
<td>0.605</td>
<td>1.431</td>
<td>2.540</td>
</tr>
<tr>
<td>Cap. Stock=10</td>
<td>1.141</td>
<td>1.140</td>
<td>1.001</td>
<td>1.751</td>
<td>2.723</td>
</tr>
<tr>
<td>A1=3</td>
<td>1.595</td>
<td>2.657</td>
<td>0.600</td>
<td>2.854</td>
<td>4.014</td>
</tr>
<tr>
<td>A2=2</td>
<td>0.798</td>
<td>1.329</td>
<td>0.600</td>
<td>0.856</td>
<td>2.471</td>
</tr>
<tr>
<td>A3=2</td>
<td>0.798</td>
<td>1.329</td>
<td>0.600</td>
<td>1.427</td>
<td>1.606</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th></th>
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<th>P_2</th>
<th>P_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Export sector is less capital intensive (\alpha &lt; \delta)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base Model*</td>
<td>1.002</td>
<td>1.542</td>
<td>0.650</td>
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<td>2.830</td>
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<tr>
<td>Tariff=0.2</td>
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<td>0.645</td>
<td>2.086</td>
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<tr>
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<td>1.078</td>
<td>1.083</td>
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<td>A1=3</td>
<td>2.003</td>
<td>3.084</td>
<td>0.650</td>
<td>4.159</td>
<td>4.959</td>
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<tr>
<td>A2=2</td>
<td>1.002</td>
<td>1.542</td>
<td>0.650</td>
<td>1.248</td>
<td>2.830</td>
</tr>
<tr>
<td>A3=2</td>
<td>1.002</td>
<td>1.542</td>
<td>0.650</td>
<td>2.080</td>
<td>1.840</td>
</tr>
</tbody>
</table>

Note: Unless it is stated otherwise, the parameters of the basic model used in the numerical example are as follows: \alpha = 0.7, \delta = 0.3, \theta = 0.5, s = 0.2, \beta_1 = 0.3, \beta_2 = 0.4, \tau = 0.1, (1 - \beta_1 - \beta_2) = 0.3, \bar{K} = 6, \bar{L} = 10, P_i^* = 1.4, P^* = 1.2, A_1 = 1.5, A_2 = 1.2, A_3 = 1.3, and e = 1.

*The following parameters are changed as follows: \alpha = 0.3 and \delta = 0.7.

4. ECONOMIC POLICY, TECHNOLOGY, FACTOR ENDOWMENTS AND SECTORAL OUTPUT

The main aim of this section is to examine the implications of a small change in tariff, technology, and capital accumulation for sectoral output. A substantial amount of empirical evidence carried out at aggregate level has shown a positive relationship between openness and the level or growth of output. However, this link between openness and the aggregate level of output is rarely direct. Rather, the direct effect of openness on reallocating resources according to the comparative advantage of the country, on technological advances, and on capital accumulation translates into a higher level of output or output growth. In this sense, it is important to understand the response of sectoral output to changes in tariff, technology, and capital stock.

The rest of the section is devoted to an investigation of these responses. First, the implications of a small change in tariff will be considered for sectoral output. Then, the investigation will be repeated for technological change and capital accumulation as we did in the previous section. This section will conclude with simulation exercises.

The equilibrium values of sectoral production are given in equations (A.65) to (A.67) in the Appendix to this chapter. To find out the implications of a small change in tariffs for proportional change in sectoral outputs, we take log-derivatives of these output equations with respect to tariffs. The following equations are obtained:
Equations (4.1) to (4.3) show that while a small increase in tariff unambiguously increases the output of the non-tradable goods sector, it reduces the output of the export and capital goods sectors. Although a change in tariffs has different effects on sectoral outputs, the magnitude of these effects is proportional to the share of imported inputs in total production, \((s(1 - \beta_1 - \beta_3))\), for all sectors. As explained in the first section, the second sector produces consumption goods and the demand for its supply comes from two sources, namely consumer demand and government expenditure. The consumer’s expenditure on good 2 depends on the level of income that consumers earn. It can easily be shown that consumers’ income increases as a result of an increase in tariff for small values of \(s\) and \(\theta\).

As can be seen from equation (4.4), both the second and third term are negative in the nominator when \(\alpha < \delta\) and therefore the sign of the equation is unambiguously positive. However, if \(\alpha\) is greater than \(\delta\), the third term may be negative depending on the size of \(s\) and \(\theta\). For small values of these two parameters, the response of consumer income to change in tariffs is always positive.

The impact of a change in technology and capital stock can easily be seen in the equilibrium sectoral output equations (A.65) to (A.67) given in the Appendix to this chapter. Since the relationships between technology, capital stock and sectoral outputs are straightforward, we will not reproduce the functions here. As those equations indicate, only the own technological change matters for sectors one and two. The reason for this surprising result has already been given in the previous section. However, the level of output in the capital goods sector is affected by the technological change in that sector and sector 1. As the coefficient on sector 1’s technology indicates, technological change in sector one affects the output in sector 3 because it eases the balance of payments constraint.

The implication of a small increase in capital stock for output levels is readily observed from equations (A.65) to (A.67). Simply, an increase in capital stock always leads to an increase in production. Moreover, the increased part of the capital stock is allocated among sectors according to the relative capital intensities of the sectors. A relatively less capital-intensive sector will allocate a small portion of the increment in capital stock. Nevertheless, the rate of growth of capital will always be the same among sectors and equal to the growth of rate of total capital stock.

**Simulation Exercises**

The implications of a small change in the tariff rate, the stock of capital and technology for the output and the consumption levels are given in Table 4.1. The same exercise, explained in the previous section, is
carried out for output and consumption levels in this section. As explained in the analytical part, an increase in the tariff rate reduces the production in sectors 1 and 3 but increases the level of output in the non-traded sector 2. Although the level of outputs is different, this is true for a different assumption about the capital intensities of the sectors as shown in the second part of the table. While an increase in the stock of capital and own technology results in a higher level of output for all sectors, technological advances in sector 1 only affect the level of output in sector 3. As can be seen from the table, consumption levels in sectors 1 and 2 follow the same pattern as the production in these sectors.

Table 4.1 The effect of a change in tariff rate, technology and capital stock on the level of output and consumption

<table>
<thead>
<tr>
<th>$\alpha$ &gt; $\delta$</th>
<th>Output $Y_1$</th>
<th>Output $Y_2$</th>
<th>Output $Y_3$</th>
<th>Consump-$C_1$</th>
<th>Consump-$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base Model</strong></td>
<td>5.178</td>
<td>4.531</td>
<td>1.291</td>
<td>4.557</td>
<td>4.471</td>
</tr>
<tr>
<td>Tariff=0.2</td>
<td>5.134</td>
<td>4.576</td>
<td>1.258</td>
<td>4.563</td>
<td>4.464</td>
</tr>
<tr>
<td>Cap. Stock=10</td>
<td>7.404</td>
<td>5.282</td>
<td>1.675</td>
<td>6.516</td>
<td>5.211</td>
</tr>
<tr>
<td>A2=2</td>
<td>5.178</td>
<td>7.552</td>
<td>1.291</td>
<td>4.557</td>
<td>7.451</td>
</tr>
<tr>
<td>A3=2</td>
<td>5.178</td>
<td>4.531</td>
<td>1.986</td>
<td>4.557</td>
<td>4.471</td>
</tr>
</tbody>
</table>

Export sector is less capital intensive ($\alpha$ > $\delta$)

| **Base Model** 7 | 6.256        | 3.757        | 1.362        | 5.505        | 3.706        |
| Tariff=0.2       | 6.202        | 3.793        | 1.327        | 5.513        | 3.700        |
| A1=3             | 12.51        | 3.757        | 1.676        | 11.01        | 3.706        |
| A2=2             | 6.256        | 6.261        | 1.362        | 5.505        | 6.177        |
| A3=2             | 6.256        | 3.757        | 2.095        | 5.505        | 3.706        |

Note: Unless it is stated otherwise, the parameters of the basic model used in the numerical example are as follows: $\alpha = 0.7$, $\delta = 0.3$, $\theta = 0.5$, $s = 0.2$, $\beta_1 = 0.3$, $\beta_2 = 0.4$, $\tau = 0.1$, $(1 - \beta_1 - \beta_2) = 0.3$, $K = 6$, $L = 10$, $P_1^* = 1.4$, $P^* = 1.2$, $A_1 = 1.5$, $A_2 = 1.2$, $A_3 = 1.3$, and $e = 1$.

*The following parameters are changed as follows: $\alpha = 0.3$ and $\delta = 0.7$.

5. CHANGE IN ECONOMIC POLICY AND INTERNATIONAL TRADE

In this section, the impact of economic reform on exports and imports will be discussed. Exports and imports have been at the centre of arguments on economic reform. It is argued that openness increases foreign exchange availability by encouraging exports and hence increases the availability of imports. Especially for LDCs, imported inputs are essential to carry out production. In our formal model, imported inputs enter into the production of capital goods to show the effect of trade restrictions on the general level of output in the economy. As shown below, an increase in tariff rates reduces the amount of imported inputs and thereby capital goods production. The latter has important implications for the rate of growth of the economy. In what follows, as in previous sections, the effect of a small change in tariff, technology and capital accumulation will be examined for export and import functions. Then simulation exercises will be provided.

Exports are given in equation (3.1.7) ($X = Y_1 - C_1$) in Table 3.1 as the production of sector 1 which is not consumed domestically. Under the small country assumption, a country can export as much as it wishes at constant world prices. In our model, the amount of exports is determined by the interaction between domestic demand for good 1 and the balance of payments restrictions at exogenously given world prices. In this sense, the export function given above can be regarded as the supply function of exports rather than the demand function. Therefore, the implications of a change in tariff rate, technology, and capital stock for exports are the combination of these effects on production and the domestic consumption of good 1. In the previous section, it has already been shown that while an increase in the tariff rate unambiguously reduces production in this sector, technology and capital stock
unambiguously increase the production in this sector. This negative effect will be reinforced by an 
increase in domestic consumption of good 1 because an increase in tariffs increases consumer income. 
From this discussion, it is clear that change in the tariff rate unambiguously decreases the level of exports 
in the model. The effect of capital stock and technology, however, always has a positive effect on 
exports.

The import demand that is given in equation (3.1.16) in Table 3.1 stems from the producers’ profit 
maximisation problem. As explained before, capital goods are produced using capital, labour and 
imported inputs. Therefore, demand for imports is represented as a conditional demand function for a 
given values of wage, rental rate, own price and the level of output in sector 1. The implications of 
change in the tariff rate, technology and capital stock for imported inputs can be found by taking 
derivatives of the equilibrium import function with respect to tariff, technology, and capital stock 
respectively. Taking the log-derivative of equation (A.59) in the Appendix to this chapter, we obtain:

\[
\frac{1}{M} \frac{\partial M}{\partial \tau} = \frac{s(1 - \beta_1 - \beta_2)(\alpha - \delta)(2\alpha - 1) - (1 + \tau)}{(1 + \tau)^2} < 0
\]

Equation (5.1) indicates that a small change in tariff rate unambiguously decreases imports because the 
first term in the nominator is always smaller than one. If the export sector is less capital-intensive than 
sector 2 (which is \( \alpha < \delta \)), then the negative effect of tariffs on import demand will be further 
reinforced. Intuitively, when the export sector is more capital intensive relative to sector 2, the 
proportional change in consumer income will be higher from equation (4.4). This will restrict the 
negative effect of tariffs on production in sector 3 and hence on imported inputs.

The impact of technological change and the capital stock on import demand can easily be seen 
from equation (A.59). As the equation shows, import demand is positively affected by an increase in the 
technology of the export sector and capital stock. This is due to the effect of \( P_1 \) on the determination of 
weight and rental rates as explained in the first section. However, the technological change in sector 3, has 
no effect on demand for imported inputs because increased production in sector 3 due to change in its 
technology, will be compensated for by an equivalent reduction in its price by keeping demand for the 
factors of production constant. These points will be clarified by numerical examples given in the next 
subsection.

<table>
<thead>
<tr>
<th>( \alpha &gt; \delta )</th>
<th>Exports-X</th>
<th>Output ( Y_1 )</th>
<th>Consump-( C_1 )</th>
<th>Imports-M</th>
<th>BOP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Model</td>
<td>0.621</td>
<td>5.178</td>
<td>4.557</td>
<td>0.725</td>
<td>0.000</td>
</tr>
<tr>
<td>Tariff=0.2</td>
<td>0.570</td>
<td>5.134</td>
<td>4.563</td>
<td>0.665</td>
<td>0.000</td>
</tr>
<tr>
<td>Cap. Stock=10</td>
<td>0.888</td>
<td>7.404</td>
<td>6.516</td>
<td>1.037</td>
<td>0.000</td>
</tr>
<tr>
<td>A1=3</td>
<td>1.243</td>
<td>10.356</td>
<td>9.114</td>
<td>1.450</td>
<td>0.000</td>
</tr>
<tr>
<td>A2=2</td>
<td>0.621</td>
<td>5.178</td>
<td>4.557</td>
<td>0.725</td>
<td>0.000</td>
</tr>
<tr>
<td>A3=2</td>
<td>0.621</td>
<td>5.178</td>
<td>4.557</td>
<td>0.725</td>
<td>0.000</td>
</tr>
</tbody>
</table>

| Export sector is less capital intensive ( \( \alpha < \delta \) ) |
|----------------|------------|-----------------|-----------------|-----------|-----|
| Base Model*    | 0.751      | 6.256           | 5.505           | 0.876     | 0.000|
| Tariff=0.2     | 0.689      | 6.202           | 5.513           | 0.804     | 0.000|
| Cap. Stock=10   | 0.875      | 7.292           | 6.417           | 1.021     | 0.000|
| A1=3           | 1.501      | 12.512          | 11.011          | 1.752     | 0.000|
| A2=2           | 0.751      | 6.256           | 5.505           | 0.876     | 0.000|
| A3=2           | 0.751      | 6.256           | 5.505           | 0.876     | 0.000|

Note: Unless it is stated otherwise, the parameters of the basic model used in the numerical example are as 
follows: \( \alpha = 0.7 \), \( \delta = 0.3 \), \( \theta = 0.5 \), \( s = 0.2 \), \( \beta_1 = 0.3 \), \( \beta_2 = 0.4 \), \( \tau = 0.1 \), 
\( (1 - \beta_1 - \beta_2) = 0.3 \), \( K^* = 6 \), \( L = 10 \), \( P^*_1 = 1.4 \), \( P^*_2 = 1.2 \), \( A_1 = 1.5 \), \( A_2 = 1.2 \), \( A_3 = 1.3 \), and \( e = 1 \).
*The following parameters are changed as follows: \( \alpha = 0.3 \) and \( \delta = 0.7 \).

Simulation Results
Using the same type of exercises as in previous sections, the numerical results for export and import functions are found and presented in Table 5.1. To see the interaction between consumption in sector 1 and exports, the output and consumption results for sector 1 are also added to the table. According to these results, while an increase in tariff rate from 10 percent to 20 percent reduces the level of imports and exports, technological advances in sector 1 and an increase in the stock of capital have a positive effect on both exports and imports. However, technological change in sectors 2 and 3 affects neither exports nor imports. These arguments hold when the export sector becomes less capital-intensive sector but the level of exports and imports is higher as shown in the second part of the table.

6. CONCLUSION
In this chapter, the implications of a small change in tariff rate, technology and the stock of capital are examined for prices, output, and international trade making use of a three-sector open economy general equilibrium model. Analytical results are further supported by numerical examples for each sector of the model.

The analysis carried out in this chapter has confirmed that there are complex interactions among sectors of the economy. As a result, economic policy changes may have different implications for different sectors and for the factors of production in the economy. In particular, the model given in this chapter has shown that an increase in tariff rates reduces the production in tradable sectors of the economy, and increases the production in the non-traded goods sector. It is also seen that factor rewards are closely related to the factor intensities between the export and non-traded goods sectors. This may have important cost implications for the factors of production during the reform programme.

It is important to note that the results presented in this chapter cannot be generalised because they are restricted to the particular assumptions of the model. However, we can safely argue that policy reform programmes should consider sectoral characteristics of the economy to minimise the cost of transition to an open market economy.

APPENDIX A
A THREE-SECTOR GENERAL EQUILIBRIUM OPEN ECONOMY MODEL

The main equations of the model are presented in Table 2.1 in the text. These involve the commodity market demand and unit price equations, factor markets’ demand equations and market clearing conditions in each of these markets. In this section, we, first, provide the detailed derivation of these equations and then present the market equilibrium values of endogenous variables of the model and the sectoral growth equations. To make it easier to understand, we divide the discussion on the general equilibrium system into the production and consumption components. After providing the general characteristics of the sectors, we first examine the production side of the model and then concentrate on the consumption part and then equilibrium relationships in the rest of the chapter.

As mentioned before, we assume that there are three sectors in the economy, two consumption goods sector and one capital goods sector. The first two sectors of the economy produce consumption goods using labour and capital that are produced by the capital goods sector. Furthermore, while the first consumption goods sector exports part of its production to meet with imports, the production of the second consumption goods sector is consumed domestically. The third sector produces capital goods for the other sectors as well as itself using labour and capital as well as imported import goods.

PRODUCTION:
In production part, we assume that there are many profit maximising and cost minimising firms in each of the three sectors, which produce using the following sectoral production functions:

\[
Y_i = A_i K_i^\alpha L_i^{\alpha-1} \quad (A.1)
\]

\[
Y_i = A_i K_i^\beta L_i^{\beta-1} \quad (A.2)
\]
\[ Y_i = A_i K_i^\beta L_i^\delta M^{1-\beta-\delta} \]  

(A.3)

where \( Y_i \), \( K_i \), \( L_i \) and \( M \) represent output, capital, labour and intermediate imported goods in sector \( i \) respectively. \( i = \) consumption good 1, consumption good 2 and capital good sectors.

Since it is assumed that firms maximise their profits in all sectors, firms’ maximisation problem leads to the following first order conditions,

\[ \Pi_1' = P_i Y_i - w L_i - r K_i \]

\[ \frac{\partial \Pi_1}{\partial L_i} = P_i (1-\alpha) K_i^\alpha L_i^{-\alpha} - w = 0 \]  

(A.4)

\[ \frac{\partial \Pi_1}{\partial K_i} = P_i \alpha K_i^{\alpha-1} L_i^{-\alpha} - r = 0 \]  

(A.5)

\[ \Pi_2' = P_i Y_i - w L_i - r K_i \]

\[ \frac{\partial \Pi_2}{\partial L_2} = P_i (1-\delta) K_2^\delta L_2^{-\delta} - w = 0 \]  

(A.6)

\[ \frac{\partial \Pi_2}{\partial K_2} = P_i \delta K_2^{\delta-1} L_2^{-\delta} - r = 0 \]  

(A.7)

\[ \Pi_3' = P_i Y_i - w L_i - r K_i - PM \]

\[ \frac{\partial \Pi_3}{\partial L_3} = P_i \beta_3 K_3^\beta L_3^{-\beta} M^{1-\beta-\delta} - w = 0 \]  

(A.8)

\[ \frac{\partial \Pi_3}{\partial K_3} = P_i \beta_1 K_3^{\beta-1} L_3^{\beta} M^{1-\beta-\delta} - r = 0 \]  

(A.9)

\[ \frac{\partial \Pi_3}{\partial M} = P_i (1-\beta_1 - \beta_3) K_3^\beta L_3^{\beta} M^{1-\beta-\delta} - P = 0 \]  

(A.10)

Solving these first order conditions in terms of factors of production and substituting them into production functions (A.1) to (A.3) give the following conditional factor demand equations as functions of relative factor prices and levels of output and they corresponds to the equations (2.10) to (2.16) given in Table 2.1.

\[ L_i = \frac{Y_i}{A_i} \left( \frac{w}{1-\alpha} \right)^{\alpha} \left( \frac{r}{\alpha} \right)^{\alpha} \]  

(A.11)

\[ L_2 = \frac{Y_2}{A_2} \left( \frac{w}{1-\delta} \right)^{\delta} \left( \frac{r}{\delta} \right)^{\delta} \]  

(A.12)

\[ L_3 = \frac{Y_3}{A_3} \left( \frac{w}{\beta_3} \right)^{\beta-1} \left( \frac{r}{\beta_3} \right)^{\beta} \left( \frac{P}{1-\beta_1 - \beta_3} \right)^{1-\beta-\delta} \]  

(A.13)

\[ K_1 = \frac{Y_1}{A_1} \left( \frac{w}{1-\alpha} \right)^{1-\alpha} \left( \frac{r}{\alpha} \right)^{1-\alpha} \]  

(A.14)

\[ K_2 = \frac{Y_2}{A_2} \left( \frac{w}{1-\delta} \right)^{1-\delta} \left( \frac{r}{\delta} \right)^{1-\delta} \]  

(A.15)

\[ K_3 = \frac{Y_3}{A_3} \left( \frac{w}{\beta_3} \right)^{\beta-1} \left( \frac{r}{\beta_3} \right)^{\beta} \left( \frac{P}{1-\beta_1 - \beta_3} \right)^{1-\beta-\delta} \]  

(A.16)

\[ M = \frac{Y_i}{A_i} \left( \frac{w}{\beta_3} \right)^{\beta} \left( \frac{r}{\beta_3} \right)^{\beta} \left( \frac{P}{1-\beta_1 - \beta_3} \right)^{1-\beta-\delta} \]  

(A.17)
From the zero profit assumption, factors of production share all output;
\[ P_1 Y_1 = w_1 L_1 + r_1 K_1 \] (A.18)
\[ P_2 Y_2 = w_2 L_2 + r_2 K_2 \] (A.19)
\[ P_3 Y_3 = w_3 L_3 + r_3 K_3 + PM \] (A.20)

Then substituting conditional factor demand equations (A.11) to (A.17) into zero profit equations (A.18) to (A.20) will provide relative price equations in terms of wage rental ratio. These prices represent equilibrium in producers’ side in a way that at given wage rental ratio, output prices clear the output market.

\[
P_1 = \frac{w_1^{\alpha-1}r^\alpha}{A_1} \left[ \frac{1}{\alpha(1-\alpha)^{-\alpha}} \right]
\] (A.21)
\[
P_2 = \frac{w_2^{\alpha-1}r^\alpha}{A_2} \left[ \frac{1}{\delta(1-\delta)^{-\alpha}} \right]
\] (A.22)
\[
P_3 = \frac{w_3^{\alpha-1}r^\alpha P_1^{1-\beta_1 \cdot \beta_1} \cdot \left( \frac{1}{\beta_1} \right) \left( \frac{1}{\beta_1} \right) \left( \frac{1}{1-\beta_1 - \beta_2} \right)}{A_3}
\] (A.23)

Equations (A.21) to (A.23) correspond to the equations (2.4) to (2.6) in Table 2.1 and they represent the producers’ equilibrium in commodity markets. For a small open economy, price of export good, which is \( P_1 \), is equal to the world prices and therefore it is taken as given. Using the small country assumption, the interest rate can be written as a function of wages and price of good 1 as follows,
\[
r = \left[ A \alpha^{\alpha} (1-\alpha)^{-\alpha} P_1^{\alpha-1} \right] = zw^{-\alpha}
\] (A.24)

where \( z = \left[ A \alpha^{\alpha} (1-\alpha)^{-\alpha} P_1 \right]^{\alpha}

CONSUMPTION:

In the consumption side of the general equilibrium system, we have two agents, namely consumers and the government. While consumers decide the level of consumption from each commodity solving the utility maximisation problem for a given commodity prices and income, the government’s consumption depends on the level of tariff revenue collected. Before introducing the consumers’ maximisation problem, we, first, want to give some definitions and market clearing equations, which will be helpful to find the equilibrium in consumption side of the model.

Consumer income is equal to national income because we assume that the factors of production, labour and capital belong to the consumer. This equilibrium condition can be shown as follows,
\[ Y = wL + rK \] (A.25)
\[ Y = w(L_1 + L_2 + L_3) + r(K_1 + K_2 + K_3) \] (A.26)

Equation (A.25) (which corresponds to equation 2.19 in Table 2.1) gives the value of total consumer income and equation (A.26) shows that the nominal value of domestic production is equal to consumer income. As mentioned before, part of the first consumption goods is exported and the government consume part of the second consumption good. Equations (A.27) and (A.28) shows market clearing conditions in goods market as follows,
\[ C_1 = Y_1 - X \] (A.27)
\[ C_2 = Y_2 - G \] (A.28)

where \( C_1 \) and \( C_2 \) are the amount of good one and good two consumed by consumers, \( X \) and \( G \) represent exports and government consumption respectively. The government revenue in this model stems from tariff revenue and equal to,
\[ G = \tau(PM)E \] (A.29)

where \( G \) is tariff revenue and \( \tau \) is tariff rate, and \( E \) is exchange rate.

The balance of payments constraint can be written as,
\[ P_X = PM \quad \text{and} \quad P = EP(1 + \tau) \] (A.30)
Furthermore, we assume that consumers save constant proportion of their income, s. And let us rewrite tariffs revenue that is transferred by government to the consumers in terms of proportion of factor income and represent it with $i$ .

$$\text{Saving} = s(wL + rK) \quad \text{(A.31)}$$

Normalising by $E$ ($E=1$), this equation can be rewritten as,

$$S = s(wL + rK) \quad \text{(A.32)}$$

Now, we can define the consumers’ maximisation problem. Consumers maximise the utility function subject to the budget constraint given below:

$$U = C_1^{\gamma}C_2^{\gamma} - \lambda[P C_1 + P C_2 - (1-s)(wL + rK)] \quad \text{(A.33)}$$

The first order conditions obtained from the maximisation of equation (A.33) show consumer demand for goods and equal to:

$$P C_1 = g(1-s)(wL + rK) \quad \text{(A.34)}$$

$$P C_2 = (1-\theta)(1-s)(wL + rK) \quad \text{(A.35)}$$

To find out the equilibrium relative wage, we use labour market equilibrium.

Demand functions for sector one and two are as follows:

$$P Y_1 = P C_1 + P X \quad \text{(A.36)}$$

$$P Y_2 = P C_2 \quad \text{(A.37)}$$

and we also know that:

$$P C_1 = \theta(1-s)(wL + rK) \quad \text{(A.38)}$$

$$P C_2 = (1-\theta)(1-s)(wL + rK) \quad \text{(A.39)}$$

$$P Y_1 = s(wL + rK) \quad \text{(A.40)}$$

Therefore, we can substitute equations (A.38) and (A.39) into equations (A.36) and (A.37) and we get;

$$P Y_1 = \theta(1-s)(wL + rK) + P X \quad \text{(A.41)}$$

$$P Y_2 = (1-\theta)(1-s)(wL + rK) + G \quad \text{(A.42)}$$

$$P Y_1 = s(wL + rK) \quad \text{(A.43)}$$

Now we can substitute prices into equations (A.41) to (A.43) and find equilibrium demand equations for a given wage-rental ratio. However, we should first find the equilibrium value of imports from import conditional demand function in terms of wage rental ratio. To do this we should start substituting prices from third sector. After substituting $P_3$ from equation (A.23) into (A.43), demand equations for $Y_1$ turns out to be;

$$Y_1 = \frac{s(wL + rK)A_{\beta_1 \beta_2} \beta_1 \beta_2 (1-\beta_1-\beta_2)^{1-\beta_1-\beta_2}}{w^{1-\beta_1} \beta_1 \beta_2 \beta_3} \quad \text{(A.44)}$$

Then, substituting this value into the conditional import demand function given in equation (A.17), we find demand for imports as a function of wages as follows,

$$M = \frac{s(wL + rK)(1-\beta_1-\beta_2)}{P} \quad \text{(A.45)}$$

In the same way, we are now in a position to find sectoral demand equations in terms of wage rental ratio for sector one by substituting the value of $P_1$ from equation (A.21) and the value of $M$ from equation (A.45) into equation (A.41) considering equation (A.30). For the sector two, we substitute $P_2$ from equation (A.22) into equation (A.41).

Thereby, we get the following equations:

$$Y_1 = \frac{(wL + rK)\theta(1-s) + s(1-\beta_1-\beta_2)(1/(1+\tau))}{P} \quad \text{(A.46)}$$
Now we can express conditional factor demands in terms of wage rental ratios by substituting the values of $\lambda_1$'s given above into equations (A.11) to (A.16). Then we get following labour and capital demand functions,

$$L_1 = \frac{(1-\alpha)(wL + rK)\theta(1-s) + s(1-\beta_1-\beta_2)(1/(1+r))}{w^{1-\delta}}$$ \hspace{1cm} (A.48)\\

$$L_2 = \frac{(1-\delta)(wL + rK)\theta(1-s) + s(1-\beta_1-\beta_2)(\tau/1+r)}{w}$$ \hspace{1cm} (A.49)\\

$$L_i = \frac{\beta_i s(wL + rK)}{w}$$ \hspace{1cm} (A.50)

Using the factor market clearing condition, in which labour supply is equal to labour demand, we find the equilibrium wages as shown in equation (A.52);

$$\bar{L} = L_1 + L_2 + L_i$$ \hspace{1cm} (A.51)\\

$$w = P_i z (K/L)^{\alpha} \left(\frac{\Psi}{1-\Psi}\right)^{\alpha}$$ \hspace{1cm} (A.52)

where \( z_i = (1-\alpha)^{1-\alpha}\alpha^\alpha \) and

$$\Psi = (1-\delta)(1-\lambda_1)\theta + (1-\delta)(1-\delta) + s(1-\beta_1-\beta_2)(1/1+r)((1-\alpha)(1-\delta)+1) + \beta_1 \]

Equilibrium rate of interest, then, can be found by substituting the equilibrium value of wage into equation (A.24) as follows,

$$r = P_i z (K/L)^{\alpha-1} \left(\frac{\Psi}{1-\Psi}\right)^{\alpha-1}$$

Then, the wage rental ratio will be,

$$\frac{w}{r} = \frac{K}{L} \frac{\Psi}{1-\Psi}$$

Now, we can express the equilibrium values of factors of production, labour, capital and imports as a function of exogenous variables of the model by substituting equilibrium values of wages. The resulting equation will be as follows:

$$K_1 = \frac{\alpha K (1-\lambda_1)\theta(1-s) + s(1-\beta_1-\beta_2)(1/1+r)}{(1-\Psi)}$$ \hspace{1cm} (A.53)\\

$$K_2 = \frac{\delta K (1-\lambda_1)(1-\lambda_2)(1-s) + s(1-\beta_1-\beta_2)(\tau/1+r)}{(1-\Psi)}$$ \hspace{1cm} (A.54)\\

$$K_3 = \frac{\beta_i s K}{(1-\Psi)}$$ \hspace{1cm} (A.55)

These equations can also be expressed as,

$$K_1 = \alpha K (1-\Psi)^{-1} \Omega_1$$ \hspace{1cm} (A.56)\\

$$K_2 = \delta K (1-\Psi)^{-1} \Omega_2$$ \hspace{1cm} (A.57)\\

$$K_3 = \beta_i K (1-\Psi)^{-1}$$ \hspace{1cm} (A.58)\\

$$M = \frac{P_i z K \bar{L}^{\alpha-1} (1-\lambda_1)\theta(1-s) + s(1-\beta_1-\beta_2)(\tau/1+r)}{(1-\Psi)^{\alpha}} \frac{\Psi^{\alpha-1}}{\bar{L}}$$ \hspace{1cm} (A.59)\\

$$L_1 = (1-\lambda_1)\bar{L} (\Psi)^{-1} \Omega_1$$ \hspace{1cm} (A.60)\\

$$L_2 = (1-\delta)\bar{L} (\Psi)^{-1} \Omega_2$$ \hspace{1cm} (A.61)\\

$$L_3 = \beta_i \bar{L} (\Psi)^{-1}$$ \hspace{1cm} (A.62)

where \( \Omega_1 = \left[\theta(1-s) + s(1-\beta_1-\beta_2)(1/1+r)\right] \)

\( \Omega_2 = \left[(1-\theta)(1-s) + s(1-\beta_1-\beta_2)(\tau/1+r)\right] \)
To find out sectoral growth functions, we first write capital values as per capita terms as follows;

\[
\frac{K}{L} = k, \quad \frac{M}{L} = m \quad \text{and} \quad \frac{K}{k} = k \quad i = 1,2,3
\]

\[
k_i = k \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{\Psi}{1-\Psi} \right)
\]

\[
k_z = k \left( \frac{\delta}{1-\delta} \right) \left( \frac{\Psi}{1-\Psi} \right)
\]

\[
k_s = k \left( \frac{\beta_z}{\beta_z} \right) \left( \frac{\Psi}{1-\Psi} \right)
\]

\[
m = k \left( \frac{P^{-\gamma}(1-\beta - \beta_z)}{\beta_z} \right) \left( \frac{1}{P} \right) \left( \frac{\Psi}{1-\Psi} \right)^{\gamma}
\]

Since the net increase in the stock of capital at a point in time equals gross investment less depreciation and furthermore, gross investment is equal to product of \( Y_3 \), we write the equation of motion of the economy as:

\[
K = Y_3 - kK
\]

\[
\dot{K} = A_0 K L^n M^{1-\beta - \beta_z} - kK
\]

Substituting the equilibrium values of \( K_3 \), \( L_3 \) and \( M \) from equation (A.58), (A.59) and (A.62) into equation of motion of the economy given by equation (A.63):

\[
\dot{K} = \left[ sP^{-\gamma} \left( 1 - \beta - \beta_z \right) K^xL^x - \frac{\Psi}{1-\Psi} \right] - \lambda K
\]

After dividing both sides by \( K \) and then rewriting the equation in per capita terms, the equation of motion turns out to be;

\[
\frac{\dot{k}}{k} = sK^{-\gamma} \left( \frac{P^{\gamma-1}}{1-\Psi} \right) \left( \frac{P}{P} \right) \left( \frac{1}{1-\Psi} \right) - (\lambda + n + g)
\]

\[
\lambda = \alpha(1-\beta_1 - \beta_2) + \beta_1
\]

\[
z_i = \beta_z^{\lambda} (1 - \beta - \beta_z)^{1-\beta - \beta_i}
\]

\[
z_i = (1-\alpha)^{1-\alpha}
\]

At the steady state, capital per labour will be equal to;

\[
k^* = \left[ \frac{z_i z_i^{\lambda-\beta-\beta_i}}{(\lambda + n + g) (1-\Psi)} \left( \frac{P}{P} \right) \left( \frac{1}{1-\Psi} \right) \right]^{\frac{1}{1-\alpha}}
\]

Then log-linearisation of equation (A.64) around steady state \( k^* \) and substituting the value of \( k^* \) in the equation will give the following equation of motion of total capital stock per labour,\n
\[
\frac{\dot{k}}{k} = \Lambda - (1-\Theta) \ln k^*
\]

where \( 1 - \Theta = (1-\kappa)(\lambda + n + g) \) and

\[
\Lambda = \left[(\lambda + n + g) \left( \ln P - \ln P - \kappa \ln(1-\Psi) + (\kappa - 1) \ln \Psi + \Phi \right) \right]
\]

\[
\Phi = \ln s - \ln(\lambda + n + g) + \beta_1 \ln \beta_1 + \beta_2 \ln \beta_1 + (1 - \beta_1 - \beta_2) \ln(1 - \beta_1 - \beta_2)
\]

\[
+ (1 - \beta_i - \beta_z) \alpha \ln(1 - \beta_i - \beta_z)(1 - \alpha) \ln(1 - \alpha)
\]

Since sectoral capital stocks have already been written in terms of total capital stock, and the equation of motion of total capital stock is known, it is a very straightforward task to write the growth equation for each sector as follows:
\[ \ln A_i = a_{it} + g, t \]
\[ \ln y = \ln A_i + \alpha \ln k, \]  \hspace{1cm} (A.67)
\[ \ln y_2 = \ln y_1 + \delta \ln k, \]  \hspace{1cm} (A.68)
\[ \ln y_3 = \ln A_i + \beta_1 \ln k_1 + (1 - \beta_i - \beta_j) \ln m \]  \hspace{1cm} (A.69)

Hence, sectoral equations can be written in terms of overall economy capital per labour as follows:

\[ \ln y_1 = a_1 + \alpha \ln k + \alpha \ln \left( \frac{\alpha - \Psi}{1 - \alpha - \Psi} \right) \]
\[ \frac{\dot{y}_1}{y_1} = g_1 + \alpha \frac{k}{k} \]
\[ \ln y_2 = a_2 + \delta \ln k + \delta \ln \left( \frac{\delta - \Psi}{1 - \delta - \Psi} \right) \]
\[ \frac{\dot{y}_2}{y_2} = g_2 + \delta \frac{k}{k} \]
\[ \frac{\dot{y}_3}{y_3} = g_3 + \beta_1 \frac{k}{k} + (1 - \beta_i - \beta_j) \frac{m}{m} \]
\[ \frac{\dot{y}_3}{y_3} = g_3 + \beta_1 \frac{k}{k} + (1 - \beta_i - \beta_j) \frac{\dot{k}}{k} \]

\[ \ln y_2 = g_2 + (1 - \Theta) g, t + \alpha (\lambda + n + g) \left[ - (1 + r)(1 - \beta_i - \beta_j) - \ln (1 - \Psi) \right] + \alpha (\lambda + n + g) \Phi + (1 - \Theta) \left[ \alpha \ln (\alpha / 1 - \alpha) \right] + \Theta \ln y_{2i-1} \]  \hspace{1cm} (A.70)
\[ \ln y_3 = g_3 + (1 - \Theta) g, t + \delta (\lambda + n + g) \left[ - (1 + r)(1 - \beta_i - \beta_j) - \ln (1 - \Psi) \right] + \delta (\lambda + n + g) \Phi + (1 - \Theta) \left[ \delta \ln (\delta / 1 - \delta) \right] + \Theta \ln y_{3i-1} \]  \hspace{1cm} (A.71)
\[ \ln y_5 = g_1 + (1 - \Theta) g, t + (1 - \beta_2) (\lambda + n + g) \left[ -(1 + r)(1 - \beta_i - \beta_j) (2 - \kappa) \right] - \ln (1 - \Psi) + \Phi + (1 - \Theta) (1 - \beta_i) \Sigma + \Theta \ln y_{5i-1} \]  \hspace{1cm} (A.72)
\[ \Sigma = \beta_i \ln \beta_i / \beta_{ji} + (1 - \beta_i - \beta_j) \ln \left( \frac{(1 - \beta_i - \beta_j)}{\beta_j} \right) \]

Equations (A.70) to (A.72) present the short-run sectoral growth equations. The examination of these equations shows that the level of sectoral output is the function of tariff rates and a small change in tariffs affects the level of output according to the share of imports that particular sector uses in production.

**BIBLIOGRAPHY**


