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COMMENTS ON PARTIAL ADJUSTMENT WITHOUT APOLOGY

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"Partial Adjustment without Apology"
by
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1. Main Analysis

The recent literature on lumpy investment, irreversibility, and the costly adjustment of employment and capital stocks has yielded many new insights into the dynamics of investment and employment. This paper contributes to this literature by providing a rationalization of the partial adjustment model using a model of plant-level adjustment.

The partial adjustment model states that a firm adjusts its employment to a target level N_t^* as

$$N_t - N_{t-1} = \kappa \left[N_t^* - N_{t-1}\right], \quad 0 < \kappa < 1.$$

Using the linear-quadratic dynamic optimization approach, Sargent (1978) showed that the partial adjustment model could be rationalized in terms of the dynamic choice problem of a firm under uncertainty which faces quadratic costs of adjustment and a second-order Taylor approximation to a standard concave production in employment.

The linear-quadratic dynamic optimization approach is appealing because

- it provides a simple and intuitive pedagogical approach. In my teaching of Graduate Macroeconomics at the University of Minnesota in the period 1984-1990, it was one of the bread-and-butter topics that was used to introduce students to the principles of dynamic optimization.

- second, the LQ approach shows the impact of lagged variables and forward-looking expectations of firms' decisions.

However, as King and Thomas enumerate, the simple partial adjustment model– whether it is augmented with forward-looking behavior or not – is incapable of accounting for the various stylized facts that characterize firms' employment decisions. Their contribution is to generate a partial-adjustment type framework describing the behavior of aggregate employment based on a model with a continuum of production units that face differential costs of adjustment and that make discrete employment decisions.

The trick or device that King and Thomas use to generate the industry-wide law of motion for employment is to consider the industry equilibrium in terms of a social planner's problem as in Lucas and Prescott (1971), and to show the equivalence of the two approaches. Since the model of industry equilibrium is also a favorite pedagogical tool (at least in my experience), reading this paper was, in some sense, a visitation to the past.

A more substantive contribution of the paper is to introduce exogenous probabilities of adjustment, α_j, that depend on the length of time since a firm has last adjusted. This yields the representation

$$D_1 f(n_t, A_t) + E \sum_{j=1}^{J-1} \Big[[\beta(1-d)]^{j} \varphi_j \Big(D_1 f((1-d)^j n_t, A_{t+j}) - W_{t+j} \Big) | A_t, W_t \Big] = W_t,$$

where

$$\varphi_j \equiv \Pi_k^j (1-\alpha_k), \;\; j = 1, \ldots, J-1.$$

This representation displays two important properties:

- As in the standard partial adjustment based on a dynamic optimization approach, the firm's employment is *forward-looking*, incorporating expectations of future wages and productivity.

- The firm's optimal employment decision possesses a *hedging* property, arising from the fact that the firm must hire more workers than implied by the standard static labor demand model to offset future forecasted labor force departures.

If the adjustments are endogenous, then the appropriate representation is given by

$$D_1 f(n_t, A_t) + E \sum_{j=1}^{J-1} \Big[[\beta(1-d)]^{j} \varphi_{j,t+j} \Big(D_1 f((1-d)^j n_t, A_{t+j}) - W_{t+j} \Big) | A_t, W_t \Big] = W_t,$$

where

$$\varphi_{j,t+j} \equiv \Pi_k^j (1-\alpha_{k,t+k}) = \Pi_k^j (1 - G(\bar{\xi}_{k,t+k})), \;\; j = 1, \ldots, J-1.$$

In this expression, $\varphi_{j,t+j}$ is the probability that the production unit will make no adjustments in the next j periods, and $\bar{\xi}_{jt}$ denotes the random adjustment cost for which the marginal plant in the j^{th} time-since-adjustment group finds it worthwhile to adjust.

The individual labor demands can be aggregated to yield a market labor demand as

$$N_t = n_t^* \sum_{j=1}^{J} \theta_{j,t} \alpha_{j,t} + \sum_{j=1}^{J-1} \theta_{j,t} (1-\alpha_{j,t}) d^j n_{t-j}^*,$$

where n_t^* is the target employment for those firms which adjust in period t, $\theta_{j,t}$ is the fraction of production units that begin the period as having adjusted j periods ago. In contrast to the standard partial adjustment model, the lag weights depend on firm and aggregate state variables, and hence, reflect changes in economic conditions on the optimal adjustment of employment at the industry or aggregate level. The numerical results reveal the importance of the extensive margin, namely, the ability of firms to adjust their employment levels, and also show the impact of aggregate conditions on changes in employment.

1.1 Fixed Costs and Irreversibility

The approach that the authors use is based on the (s, S) model with stochastic adjustment costs as proposed by Engel and Caballero (1999) in their study of manufacturing investment. The Caballero-Engel (1999) model abstracts from irreversibility. Irreversibility may also be important for employment decisions, that is, once a firm creates a job by hiring workers, it may find it difficult to fire them. This would induce an *option value of waiting* in addition to the *hedging motive* that firms face when confronted with the anticipated attrition of their labor force.

We introduce the model with fixed costs and irreversibility following the analysis in Demers, Demers, and Altug (2003). Assume that the firm faces a fixed cost of investing, f_t which is proportional to the firm's stock of capital. As a result, the total cost of investing becomes $f_t K_t + p_t^k I_t$. The firm's after-tax cash flow at time t, R_t, is defined as

$$R(K_t, h_t, p_t^k, f_t) = \Pi(K_t, h_t) - f_t K_t - p_t^k I_t \tag{1}$$

We assume that the demand shock α_t is serially correlated and that there are no other sources of uncertainty. The optimization problem of the firm can now be stated as:

$$\begin{aligned} V(K_t, h_t, h_t^k, f_t) &= \max_{I_t} \{R(K_t, h_t, p_t^k, f_t) + \beta E_t V(K_{t+1}, h_{t+1}, p_{t+1}^k, f_{t+1})\} \\ &\quad \text{subject to } I_t \geq 0 \textit{ and } K_{t+1} = (1-\delta)K_t + I_t \end{aligned} \tag{2}$$

The first order condition becomes:

$$\begin{aligned} & -p_t^k + \beta E_t V_K(K_{t+1}, h_{t+1}, p_{t+1}^k, f_{t+1}) = 0 \\ & \qquad \text{if } \beta E_t V_K((1-\delta)K_t, h_{t+1}, p_{t+1}^k, f_{t+1}) > p_t^k \end{aligned} \tag{3}$$

$$\begin{aligned} & K_{t+1} = (1-\delta)K_t \\ & \qquad \text{if } \beta E_t V_K((1-\delta)K_t, h_{t+1}, p_{t+1}^k, f_{t+1}) \leq p_t^k \end{aligned} \tag{4}$$

The shadow value of capital can be found as:

$$\begin{aligned} V_K(K_{t+1}, h_{t+1}, p_{t+1}^k, f_{t+1}) &= \Pi_K(K_{t+1}, h_{t+1}) - f_{t+1} \\ &\quad + (1-\delta)\min[p_{t+1}^k, \beta E_{t+1} V_K((1-\delta)K_{t+1}, h_{t+2}, p_{t+2}^k, f_{t+2})]. \end{aligned} \tag{5}$$

An examination of (5) reveals that when solving the first order condition for the optimal capital stock and investment the firm will need to take into account the impact of an additional unit of capital, K_{t+1}, on future fixed costs. In other words, the expected marginal value of capital is reduced by fixed costs at $t+1$ as well as in future periods.

Define α_t^H such that

$$p_t^k = \beta \int_0^\infty V_K((1-\delta)K_t, h_{t+1}, p_{t+1}^k, f_{t+1}) dG^\alpha(\alpha_{t+1}|\alpha_t^H).$$

Assuming $\alpha_t \geq \alpha_t^H$ the firm's desired stock of capital given irreversibility will be determined by the first order condition as before. That is, $p_t^k = \beta E_t V_K(K_{t+1}, h_{t+1}, p_{t+1}^k)$

where V_K takes into account the future fixed costs which an additional unit of capital entails. However, the firm will only invest when another condition is satisfied. Investment will take place and the firm will adjust its capital stock if the difference in the expected value of the firm achieved by investing exceeds the total cost of investing including the fixed cost:

$$\beta E_t V((1-\delta)K_t + I_t^*, h_{t+1}, p_{t+1}^k) - \beta E_t V((1-\delta)K_t, h_{t+1}, p_{t+1}^k) \geq f_t K_t + p_t^k I_t^*. \quad (6)$$

Hence, if this latter (value matching) condition does not hold the firm will not invest even though positive investment would be warranted by the first order condition. That is, a positive investment level will be chosen only if the net expected total benefits exceed the benefits of not investing. Therefore, the solution to the problem involves finding α_t^H and I_t^* such that (3)- (4) and (6) hold simultaneously. This framework leads to a standard (S, s) type of policy. The zone of inaction is thus larger for lumpy and irreversible investment than for irreversibility without fixed costs. The capital stock will be adjusted infrequently and by sizable amounts.[1]

1.2 Extensions

Much of the literature on the dynamic adjustment of employment and investment decisions has been done for the US economy. However, many new issues arise once one starts training one's eye on the problems of transition and emerging-market economies. Here are some facts to motivate some extensions.

- A significant degree of irreversibility may characterize employment decisions. For example, a recent study by Belke and Seltzer (2003) demonstrates that for Central and Eastern European economies exchange rate uncertainty combined with irreversibility can impede job creation and hasten job destruction.

- Many emerging-market economies have substantial underground sectors, suggesting that firms face an additional margin in terms of whether to locate in the formal versus informal sector.

- Many emerging-market economies are characterized by openness, and trade measured as the sum of their exports plus imports often accounts for a significant fraction of GDP.

One of the extensions that could be considered is the separate modelling of job creation versus job destruction. It is well-known that the dynamics of job creation differs from the dynamics of job destruction. In this paper, there is only exogenous attrition in employment. Thus, it would be interesting to examine whether the approach used in the paper could be extended to modelling job creation versus job destruction separately.

How would the analysis of employment adjustments change if the existence of an underground sector were taken into account? Thus, instead of deciding to create new

[1]Heterogeneity can be introduced by assuming that firms face different fixed costs. This would allow for aggregation.

employment or not, the firm would have three decisions, whether to locate in the formal sector, in the informal sector, or not to adjust at all? Presumably, the extent of tax provisions or the tax burden would be one of the important determinants of new employment creation.

Much employment in emerging-market economies is related to export performance. Thus, employment dynamics is partly driven by export dynamics. The export decision is typically accompanied by uncertainty, plant-level heterogeneity, and sunk entry costs. (See, for example, Das, Roberts, and Tybout (2001).)

A final point that comes to mind, not necessarily related to these points, is the impact of market structure on employment decisions. For example, do firms with greater market power have lumpier employment decisions? How would one extend the model at hand to allow for differences in market power across firms? What would be the impact of market power on employment dynamics?

References

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